Rate-Based Transition Systems and Stochastic Process Algebras

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Outline...

1. Motivations
2. Rate-based Transition Systems
3. Stochastic CSP: PEPA
4. Stochastic CCS: StoCCS
5. Conclusions and Future Directions

Outline. . .

Motivations. . .

A number of stochastic process algebras have been proposed in the last two decades. These are based on:

1. Labeled Transition Systems (LTS)
   - for providing compositional semantics of languages
   - for describing *qualitative properties*
2. Continuous Time Markov Chains (CTMC)
   - for analysing *quantitative properties*

Motivations. . .

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Semantics of these calculi have been given by variants of the Structured Operational Semantics (SOS) approach but:

- there is no general framework for modelling the different formalisms
- it is rather difficult to appreciate differences and similarities of such semantics.

Stochastic Process Algebras - incomplete list

- TIPP (N. Glotz, U. Herzog, M. Rettelbach - 1993)
- Stochastic π-calculus (C. Priami - 1995, later with P. Quaglia)
- PEPA (J. Hillston - 1996)
- EMPA (M. Bernardo, R. Gorrieri - 1998)
- IMC (H. Hermanns - 2002)
- ...
- STOKLAIM
- MarCaSPiS
- ...

More will come ... Besides qualitative aspects of distributed systems it more and more important that performance and dependability be addressed to deal with issues related to quality of service when:

- components interact over networks where failures are likely;
- congestion may cause unpredictable delays.

Common ingredients of Stochastic PA

- It is assumed that action execution takes time
  (not for IMC, there rates and actions are split)
- Execution times is described by means of random variables
- Random Variables are assumed to be exponentially distributed
- Random Variables are fully characterised by their rates.

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If X is *exponentially distributed* with *parameter* $\lambda \in \mathbf{R}_{>0}$:

- $\mathbb{P}\{X \leq d\} = 1 - e^{-\lambda \cdot d}$, for $d \geq 0$
- The average duration of X is $\frac{1}{\lambda}$; the variance of X is $\frac{1}{\lambda^2}$
- *Memory-less*: $\mathbb{P}\{X \leq t + d \mid X > t\} \; = \; \mathbb{P}\{X \leq d\}$

Continuous Time Markov Chains

Continuous Time Markov Chains are a successful mathematical framework for modeling and analysing performance and dependability of systems

CTMCs come with

- Well established Analysis Techniques
  - Steady State Analysis
  - Transient Analysis
- Efficient Software Tools:
  - Stochastic Timed/Temporal Logics
  - Stochastic Model Checking

A CTMC is a pair $(\mathcal{S}, \mathbf{R})$

- $\mathcal{S}$: a countable set of states
- $\mathbf{R} : \mathcal{S} \times \mathcal{S} \rightarrow \mathrm{R}_{\geq 0}$, the rate matrix

Stochastic process calculi

A Continuous Time Markov Chain (CTMC) is associated to each term of the process algebras. CTMC are thus used for defining the stochastic behaviour of processes.

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Process Calculi:

$$\alpha.P + \alpha.P = \alpha.P$$

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Stochastic Process Calculi:

$$\alpha^{\lambda}.P + \alpha^{\lambda}.P = \alpha^{2\lambda}.P$$

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Outline...

Semantics of stochastic process calculi

We introduce a variant of Rate Transition Systems (RTS), proposed by Klin and Sassone, and use them for defining stochastic behaviour of processes. We do rely on the *apparent rate* approach by by Hillston to model process composition.

Like most of the previous attempts we take a two step approach: For a given term, say T, we define an enriched LTS and then use it to determine the CTMC to be associated to T.

There are however two distinguishing aspects of our work:

- Differently from Klin and Sassone, our variant of RTS associates terms and actions to functions from terms to rates
- We generalize the *apparent rate* approach, originally developed by Hillston for multi-party synchronisation (à la CSP), to binary synchronisation (à la CCS)

Semantics of stochastic process calculi

Stochastic semantics of process calculi is defined by means of a transition relation $\longrightarrow$ that associates to a pair (P, α) - consisting of process and an action - a total function $(\mathscr{P}, \mathscr{Q}, \ldots)$ that assigns a non-negative real number to each process of the calculus. Value 0 is assigned to unreachable processes.

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$P \xrightarrow{\alpha} \mathscr{P}$ means that, for a generic process Q:

- if $\mathscr{P}(Q) = x$ ($\neq 0$) then Q is reachable from P via the execution of α with rate or weight x
- if $\mathscr{P}(Q) = 0$ then Q is not reachable from P via α

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We have that if $P \xrightarrow{\alpha} \mathscr{P}$ then

- $\oplus\mathscr{P} = \sum_Q \mathscr{P}(Q)$ represents the total rate/weight of α in P.

Rate transition systems

Definition (Rate Transition Systems)

A rate transition system is a triple $(S, A, \longrightarrow)$ where:

- S is a set of states;
- A is a set of transition labels;
- $\rightarrow \subseteq S \times A \times [S \rightarrow \mathbb{R}_{\geq 0}]$

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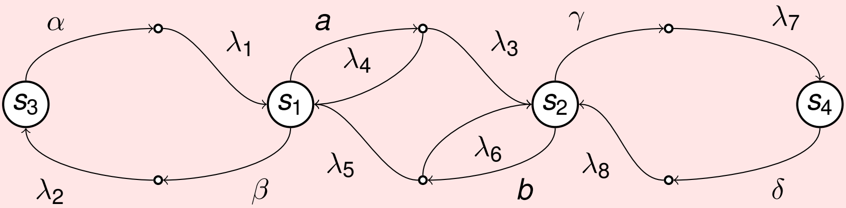


Rate transition systems

Notations:

- RTS will be denoted by $\mathcal{R}$, $\mathcal{R}_1$, $\mathcal{R}'$, ...,
- Elements of $[S \rightarrow \mathbf{R}_{\geq 0}]$ are denoted by $\mathscr{P}, \mathscr{Q}, \mathscr{R}, \ldots$
- $[s_1 \mapsto v_1, \ldots, s_n \mapsto v_n]$ denotes the function associating v_i to s_i and 0 to all the other states.
- $\emptyset$ denotes the constant function 0.
- χ_s stands for $[s \mapsto 1]$.
- $\mathscr{P} + \mathscr{Q}$ denotes the function $\mathscr{R}$ such that: $\mathscr{R}(s) = \mathscr{P}(s) + \mathscr{Q}(s)$.
- $\mathscr{P} \cdot \frac{x}{y}$ denotes the function $\mathscr{R}$ such that: $\mathscr{R}(s) = \mathscr{P}(s) \cdot \frac{x}{y}$ if $y \neq 0$, and $\emptyset$ if $y = 0$.

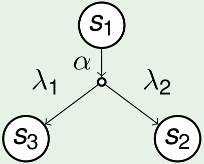
Rate transition systems

Definition

Let $\mathcal{R} = (S, A, \rightarrow)$ be an RTS, then:

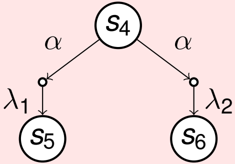
- $\mathcal{R}$ is *fully stochastic* if and only if for each $s \in S$, $\alpha \in A$, $\mathscr{P}$ and $\mathscr{Q}$ we have: $s \xrightarrow{\alpha} \mathscr{P}, s \xrightarrow{\alpha} \mathscr{Q} \Longrightarrow \mathscr{P} = \mathscr{Q}$
- $\mathcal{R}$ is *image finite* if and only if for each $s \in S$, $\alpha \in A$ and $\mathscr{P}$ such that $s \xrightarrow{\alpha} \mathscr{P}$ we have: $\{s' | \mathscr{P}(s') > 0\}$ is finite

A fully stochastic RTS. . .



. . . leads to CTMC.

General RTS. . .



. . . leads to CTMDP.

From RTS to CTMC...

Reachable Sets of States

For sets $\mathcal{S}' \subseteq \mathcal{S}$ and $A' \subseteq A$, the set of derivatives of $\mathcal{S}'$ *through* A', denoted $Der(\mathcal{S}', A')$, is the smallest set such that:

- $\mathcal{S}' \subseteq Der(\mathcal{S}', A')$,
- if $s \in Der(\mathcal{S}', A')$ and there exists $\alpha \in A'$ and $\mathscr{Q} \in \Sigma_{\mathcal{S}}$ such that $s \xrightarrow{\alpha} \mathscr{Q}$ then $\{s' \mid \mathscr{Q}(s') > 0\} \subseteq Der(\mathcal{S}', A')$

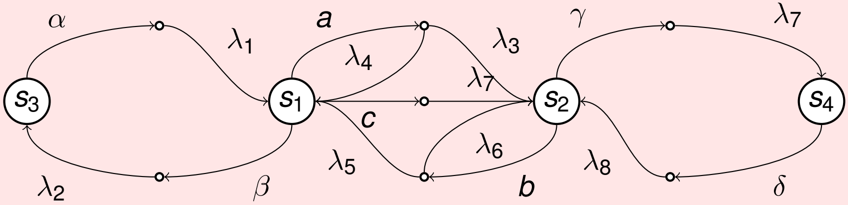
Mapping $(\mathcal{S}, A, \rightarrow)$ into $(Der(\mathcal{S}', A'), \mathbf{R})$

Let $\mathcal{R} = (\mathcal{S}, A, \rightarrow)$ be a *fully stochatics* RTS, for $\mathcal{S}' \subseteq \mathcal{S}$, the CTMC of $\mathcal{S}'$, when one considers only actions $A' \subseteq A$ is defined as $CTMC[\mathcal{S}', A'] \stackrel{def}{=} (Der(\mathcal{S}', A'), \mathbf{R})$ where for all $s_1, s_2 \in Der(\mathcal{S}', A')$:

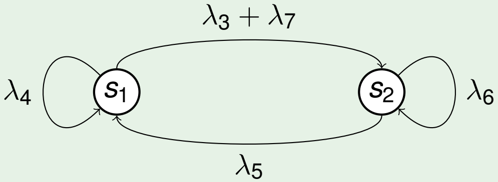
$$\mathbf{R}[s_1, s_2] \stackrel{def}{=} \sum_{\alpha \in A'} \mathscr{P}^{\alpha}(s_2) \qquad \text{with } s_1 \xrightarrow{\alpha} \mathscr{P}^{\alpha}.$$

An example...

$(\{s_1, s_2, s_3, s_4\}, \{\alpha, \beta, \gamma, \delta, a, b, c\}, \rightarrow)$



$CTMC[\{s_1, s_2\}, \{a, b, c\}]$



Strong Markovian bisimilarity

Definition

Given a generic CTMC $(S, \mathbf{R})$

- An equivalence relation $\mathcal{E}$ on S is a Markovian bisimulation on S if and only if for all $(s_1, s_2) \in \mathcal{E}$ and for all equivalence classes $C \in S_{/\mathcal{E}}$ the following condition holds: $\mathbf{R}[s_1, C] \leq \mathbf{R}[s_2, C]$.
- Two states $s_1, s_2 \in S$ are strong Markovian bisimilar, written $s_1 \sim_M s_2$, if and only if there exists a Markovian bisimulation $\mathcal{E}$ on S with $(s_1, s_2) \in \mathcal{E}$.

Rate aware bisimulation

Definition (Rate Aware Bisimilarity)

Let $\mathcal{R} = (\mathcal{S}, A, \rightarrow)$ be a RTS:

- An equivalence relation $\mathcal{E} \subseteq \mathcal{S} \times \mathcal{S}$ is a *rate aware* bisimulation if and only if, for all $(s_1, s_2) \in \mathcal{E}$, and $\underline{S} \in \mathcal{S}_{/\mathcal{E}}$, and for all α and $\mathscr{P}$:

$$s_1 \xrightarrow{\alpha} \mathscr{P} \Longrightarrow \exists \mathscr{Q} : \; s_2 \xrightarrow{\alpha} \mathscr{Q} \wedge \mathscr{P}(\underline{S}) = \mathscr{Q}(\underline{S})$$

- Two states $s_1, s_2 \in S$ are *rate aware bisimilar* $(s_1 \sim s_2)$ if there exists a rate aware bisimulation $\mathcal{E}$ such that $(s_1, s_2) \in \mathcal{E}$.

Notice that *rate aware bisimilarity* and *strong bisimilarity* coincide when one does not take into account rates.

Theorem

Let $\mathcal{R} = (\mathcal{S}, A, \longrightarrow)$, *for each* $A' \subseteq A$ *and for each* $s_1, s_2 \in \mathcal{S}$ *and* $(S, \mathbf{R}) = CTMC[\{s_1, s_2\}, A']$*:* $s_1 \sim s_2 \Longrightarrow s_1 \sim_M s_2$

Outline. . .

PEPA: Performance Process Algebra

- In PEPA systems are be described as interactions of *components* that may engage in *activities*
  - Components reflect the behaviour of relevant parts of the system, while activities capture the actions that the components perform.
- Each PEPA activity consists of a pair (α, λ) where:
  - α symbolically denotes the performed action;
  - $\lambda > 0$ is the rate of the negative *exponential* distribution.
- If $\mathcal{A}$ is a set of *actions*, ranged over by $\alpha, \alpha', \alpha_1, \ldots$, then $\mathcal{P}_{PEPA}$ is the set of process terms $P, P', P_1, \ldots$ defined according to the following grammar

$$P ::= (\alpha, \lambda).P \mid P + P \mid P \mid\mid_L P \mid P/L \mid A$$

PEPA Stochastic semantics…

$$\frac{}{(\alpha,\lambda).P \xrightarrow{\alpha} [P \mapsto \lambda]} \ (\textsc{Act})$$

$$\frac{\alpha \neq \beta}{(\alpha,\lambda).P \xrightarrow{\beta} \emptyset} \ (\emptyset\text{-}\textsc{Act})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q}}{P + Q \xrightarrow{\alpha} \mathscr{P} + \mathscr{Q}} \ (\textsc{Sum})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \notin L}{P \mid\mid_L Q \xrightarrow{\alpha} \mathscr{P} \mid\mid_L \chi_Q + \chi_P \mid\mid_L \mathscr{Q}} \ (\textsc{Int})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \in L}{P \mid\mid_L Q \xrightarrow{\alpha} \mathscr{P} \mid\mid_L \mathscr{Q} \cdot \frac{\min\{\oplus\mathscr{P}, \oplus\mathscr{Q}\}}{\oplus\mathscr{P} \cdot \oplus\mathscr{Q}}} \ (\textsc{Coop})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad \alpha \notin L}{P/L \xrightarrow{\alpha} \mathscr{P}/L} \ (\text{P-}\textsc{Hide})$$

$$\frac{\alpha \in L}{P/L \xrightarrow{\alpha} \emptyset} \ (\emptyset\text{-}\textsc{Hide})$$

$$\frac{P \xrightarrow{\tau} \mathscr{P}_\tau \quad \forall \alpha \in L. P \xrightarrow{\alpha} \mathscr{P}_\alpha}{P/L \xrightarrow{\tau} \mathscr{P}_\tau/L + \sum_{\alpha \in L} \mathscr{P}_\alpha/L} \ (\textsc{Hide})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad A \triangleq P}{A \xrightarrow{\alpha} \mathscr{P}} \ (\textsc{Call})$$

PEPA Stochastic semantics…

Prefixes and Sums

$$\frac{}{(\alpha,\lambda).P \xrightarrow{\alpha} [P \mapsto \lambda]} \ (\textsc{Act}) \qquad \frac{\alpha \neq \beta}{(\alpha,\lambda).P \xrightarrow{\beta} \emptyset} \ (\emptyset\text{-}\textsc{Act})$$

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An example derivation

$$\frac{}{((\alpha, \lambda_1).P_1 + (\beta, \lambda_2).P_2) + (\alpha, \lambda_3).P_3 \xrightarrow{\alpha}}$$

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$$\frac{\frac{}{(\alpha,\lambda_1).P_1 + (\beta,\lambda_2).P_2 \xrightarrow{\alpha}} \qquad (\alpha,\lambda_3).P_3 \xrightarrow{\alpha} [P_3 \mapsto \lambda_3]}{((\alpha,\lambda_1).P_1 + (\beta,\lambda_2).P_2) + (\alpha,\lambda_3).P_3 \xrightarrow{\alpha}}$$

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PEPA Stochastic semantics

Interleaving and Multiparty Synchronization

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \notin L}{P \parallel_L Q \xrightarrow{\alpha} \mathscr{P} \parallel_L \chi_Q + \chi_P \parallel_L \mathscr{Q}} \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \in L}{P \parallel_L Q \xrightarrow{\alpha} \mathscr{P} \parallel_L \mathscr{Q} \cdot \frac{\min\{\oplus\mathscr{P}, \oplus\mathscr{Q}\}}{\oplus\mathscr{P} \cdot \oplus\mathscr{Q}}}$$

- remember that χ_P is:

$$\chi_P(R) = \begin{cases} 1 & \text{if } R = P \\ 0 & \text{otherwise} \end{cases}$$

- $\mathscr{P} \parallel_L \mathscr{Q}$ denotes the function $\mathscr{R}$ such that:

$$\mathscr{R}(R) = \begin{cases} \mathscr{P}(P) \cdot \mathscr{Q}(Q) & \text{if } R = P \parallel_L Q \\ 0 & \text{otherwise} \end{cases}$$

A couple results for our PEPA semantics

Theorem

$\mathcal{R}_{PEPA}$ is fully stochastic and image finite.

Theorem

For all $P, Q \in \mathcal{P}_{PEPA}$ and $\alpha \in \mathcal{A}$ the following holds:

$$P \xrightarrow{\alpha} \mathscr{P} \wedge \mathscr{P}(Q) = \lambda > 0 \Leftrightarrow P \xrightarrow{\alpha,\lambda}_P Q$$

where $\longrightarrow_P$ stands for the transition relation defined by Hillstone in [Hil96].

Outline...

StoCCS: Stochastic CCS

STOCCS: Stochastic CCS

STOCCS is a Markovian extension of CCS where:

- *output activities* are enriched with *rates* characterizing random variables with exponential distributions, modeling their duration;
- *input activities* are equipped with *weights* characterizing the relative selection probability

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Like for PEPA , and for most of the other calculi, the CTMC for StoCCS specifications are obtained by only considering internal actions and channel interactions.

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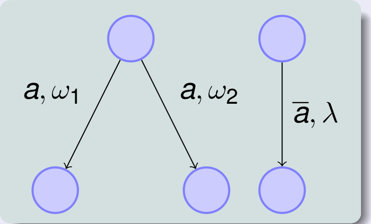

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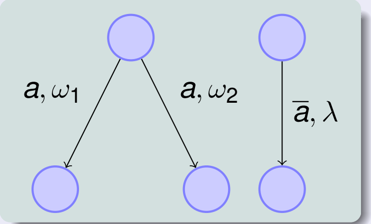
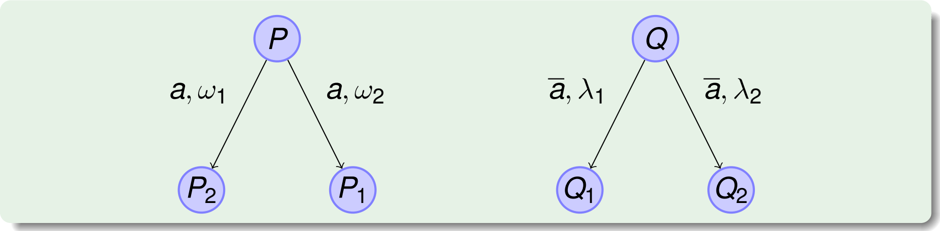


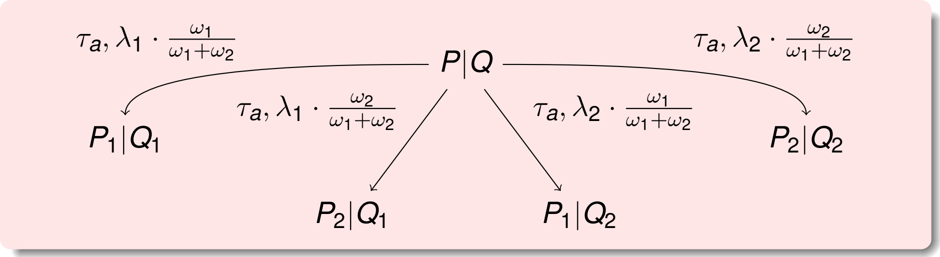
- Two synchronizations can occur with rates:

$$\lambda \cdot \frac{\omega_1}{\omega_1 + \omega_2} \qquad \lambda \cdot \frac{\omega_2}{\omega_1 + \omega_2}$$

- The overall sum of the synchronization rates is the same as the one of the output, i.e. it does not depend on the number of available (input) partners.

STOCCS: Transitions rates





STOCCS: Stochastic semantics - 1st attempt

Binary Synchronisation:

$$\frac{P \xrightarrow{\tau_a} \mathscr{P} \quad P \xrightarrow{a} \mathscr{P}_i \quad P \xrightarrow{\bar{a}} \mathscr{P}_o \quad Q \xrightarrow{\tau_a} \mathscr{Q} \quad Q \xrightarrow{a} \mathscr{Q}_i \quad Q \xrightarrow{\bar{a}} \mathscr{Q}_o}{P|Q \xrightarrow{\tau_a} \mathscr{P}|\chi_Q + \chi_P|\mathscr{Q} + \frac{\mathscr{P}_i|\mathscr{Q}_o}{\oplus \mathscr{P}_i} + \frac{\mathscr{P}_o|\mathscr{Q}_i}{\oplus \mathscr{Q}_i}}$$

Next states of $P|Q$ after τ_a, i.e. after a synchronisation over channel a, are:

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4. the next states of P after a in parallel with the next states of Q after $\bar{a}$.

StoCCS: Stochastic semantics - 1st attempt

Theorem

$\mathcal{R}_{StoCCS}$ is fully stochastic and image finite.

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The proposed semantics coincides with the one proposed by Klin and Sassone.

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Problem

The proposed semantics does not respect a standard and expected property of the CCS parallel composition.

The $|$ operator is not associative!

StoCCS: Stochastic semantics, 1st attempt

A counterexample for associativity

For instance:

$$\overline{a}^{\lambda}.P|(a^{\omega_1}.Q_1|a^{\omega_2}.Q_2) \xrightarrow{\tau_a}$$

$$(\overline{a}^{\lambda}.P|a^{\omega_1}.Q_1)|a^{\omega_2}.Q_2 \xrightarrow{\tau_a}$$

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$$\overline{a}^{\lambda}.P|(a^{\omega_1}.Q_1|a^{\omega_2}.Q_2) \xrightarrow{\tau_a} [P|(Q_1|a^{\omega_2}.Q_2) \mapsto \frac{\lambda\cdot\omega_1}{\omega_1+\omega_2}, P|(a^{\omega_1}.Q_1|Q_2) \mapsto \frac{\lambda\cdot\omega_2}{\omega_1+\omega_2}]$$

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$$(\overline{a}^{\lambda}.P|a^{\omega_1}.Q_1)|a^{\omega_2}.Q_2 \xrightarrow{\tau_a} [(P|Q_1)|a^{\omega_2}.Q_2 \mapsto \lambda, (P|a^{\omega_1}.Q_1)|Q_2 \mapsto \lambda]$$

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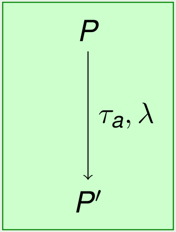
Theorem (From Klin and Sassone - KS08)

StoCCS *parallel composition is associative up-to stochastic bisimilarity if and only if the rate of a synchronisation is determined as the product of the two rates of the involved actions.*

Computing the rate of a synchronization



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$$P \xrightarrow{\tau_a,\,\lambda} P' \qquad\qquad a^{\omega}.Q \xrightarrow{a,\,\omega} Q$$

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$$P | a^{\omega}.Q \xrightarrow{\tau_a,\, \lambda'} P' | Q$$

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If $\overline{\omega}$ is the total weight of a in P:

$$\lambda' =$$

Computing the rate of a synchronization

$$P \xrightarrow{\tau_a, \lambda} P' \qquad a^{\omega}.Q \xrightarrow{a, \omega} Q \qquad P|a^{\omega}.Q \xrightarrow{\tau_a, \lambda'} P'|Q$$

If $\overline{\omega}$ is the total weight of *a* in *P*:

$$\lambda' = \lambda \cdot \frac{\overline{\omega}}{\overline{\omega} + \omega}$$

Computing the rate of a synchronization

$$\frac{P \xrightarrow{\tau_a,\,\lambda} P' \qquad a^{\omega}.Q \xrightarrow{a,\,\omega} Q}{P|a^{\omega}.Q \xrightarrow{\tau_a,\,\lambda'} P'|Q}$$

If $\overline{\omega}$ is the total weight of a in P:

$$\lambda' = \lambda \cdot \frac{\overline{\omega}}{\overline{\omega} + \omega}$$

This is the key point to guarantee associativity of parallel composition in CCS-like synchronizations.

STOCCS: stochastic semantics, 2nd attempt

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Interactions on channel a in $P|Q$ are determined by considering

- the synchronisations in P, where synchronization rates are updated for considering input in Q;

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STOCCS: stochastic semantics, 2nd attempt

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- interactions between input in P with output in Q.

STOCCS: stochastic semantics

Theorem

In StoCCS parallel composition is associative up to rate aware bisimilarity, i.e. for each P, Q and R, $P|(Q|R) \sim (P|Q)|R$

Stoπ: Stochastic π-Calculus

Input, Output and Synchronisation:

$$\frac{}{\overline{a}b^{\lambda}.P \xrightarrow{\overline{a}b} [P \mapsto \lambda]} \ (\textsc{Out}) \qquad \frac{}{a(x)^{\omega}.P \xrightarrow{ab} [P[b/x] \mapsto \omega]} \ (\textsc{In})$$

$$\frac{\begin{array}{ccc} P \xrightarrow{\tau_a(b)} \mathscr{P} & P \xrightarrow{ab} \mathscr{P}_i & P \xrightarrow{\overline{a}b} \mathscr{P}_o \\ Q \xrightarrow{\tau_a(b)} \mathscr{Q} & Q \xrightarrow{ab} \mathscr{Q}_i & Q \xrightarrow{\overline{a}b} \mathscr{Q}_o \end{array}}{P|Q \xrightarrow{\tau_a b} \frac{\mathscr{P}|Q \cdot \oplus \mathscr{P}_i}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i} + \frac{P|\mathscr{Q} \cdot \oplus \mathscr{Q}_i}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i} + \frac{\mathscr{P}_i|\mathscr{Q}_o}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i} + \frac{\mathscr{P}_o|\mathscr{Q}_i}{\oplus \mathscr{P}_i + \oplus \mathscr{Q}_i}} \ (\textsc{Sync})$$

The other rules are the expected ones.

Outline. . .

Summing Up

- We have introduced Rate Transition Systems and have used them as the basic model for defining stochastic behaviour of processes.
- We have introduced a natural notion of bisimulation over RTS that agrees with Markovian bisimulation.
- We have shown how RTS can be used to provide the stochastic operational semantics of PEPA and CCS.
- We have discussed the generalization of the approach to π-calculus and MarCaSPiS.

Future Work

- Use RTS to model other formalisms
- Use the RTS approach as general framework for modelling other PA semantics (non-deterministic, truly-concurrent, probabilitistic,...)
- Consider alternative semantics synchonisation rates:
    - based on *phase type* distributions
    - based on *Interactive Markov Chains*
- Develop tools directly for RTS rather than for CTMC.

Thank you for your attention!