Quantitative Abstraction Refinement

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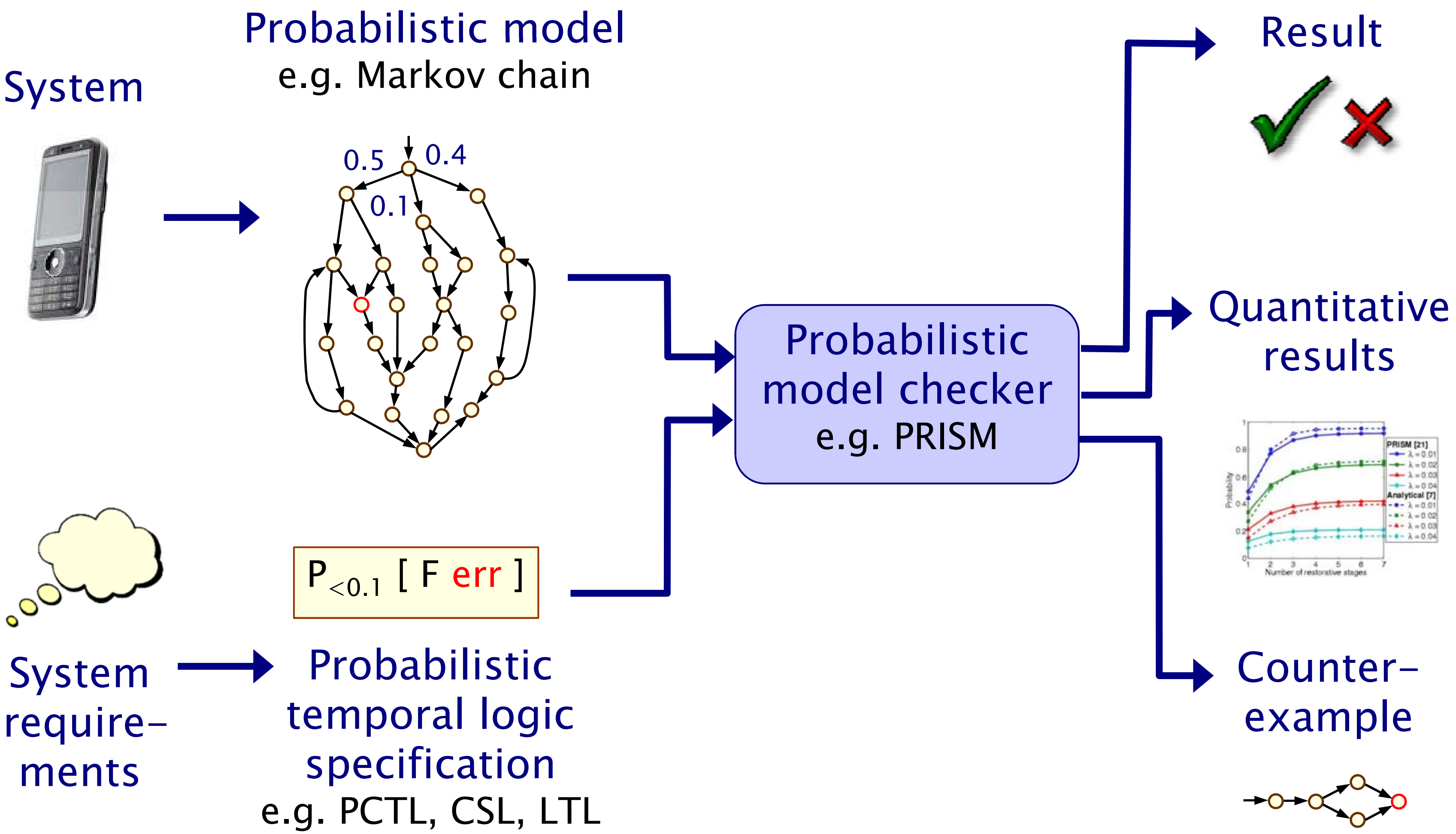
Probabilistic model checking



Overview

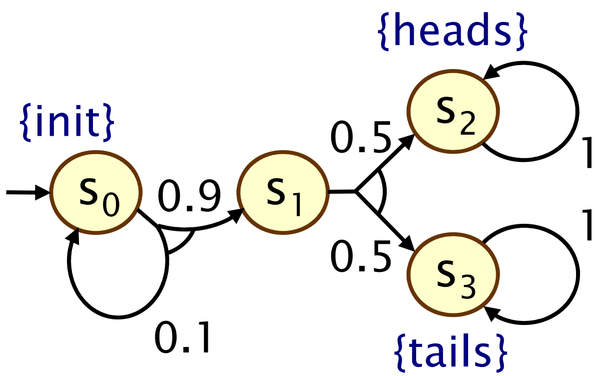
- Probabilistic model checking
  - Markov decision processes (MDPs)
  - probabilistic timed automata (PTAs)
- Abstraction for probabilistic models
  - abstractions of MDPs (stochastic two-player games)
- Quantitative abstraction refinement
  - abstraction-refinement loop
  - probabilistic model checking for PTAs
  - also: verification of probabilistic software
- Conclusions & current/future work

Probabilistic models

- Discrete-time Markov chains (DTMCs)
  - discrete states, discrete probability distributions
- Markov decision processes (MDPs)
  - discrete states, probability and nondeterminism
- Probabilistic timed automata (PTAs)
  - discrete states, probability, nondeterminism and dense time
- Continuous-time Markov chains (CTMCs)
  - discrete states, exponentially distributed delays
- And more... (CTMDPs, IMCs, LMPs, …)

Markov decision processes (MDPs)

- Model nondeterministic as well as probabilistic behaviour
  - e.g concurrency, environmental factors, under-specification, ...
- Formally, an MDP is a tuple (S, Act, Steps) where:
  - S is a set of states
  - Act is a set of actions
  - Steps : $S \times Act \rightarrow Dist(S)$ is the transition probability function



- An adversary (aka. “scheduler”or “policy”) of an MDP
  - is a resolution of the nondeterminism in the MDP
  - under a given adversary σ the behaviour is fully probabilistic

Probabilistic reachability for MDPs

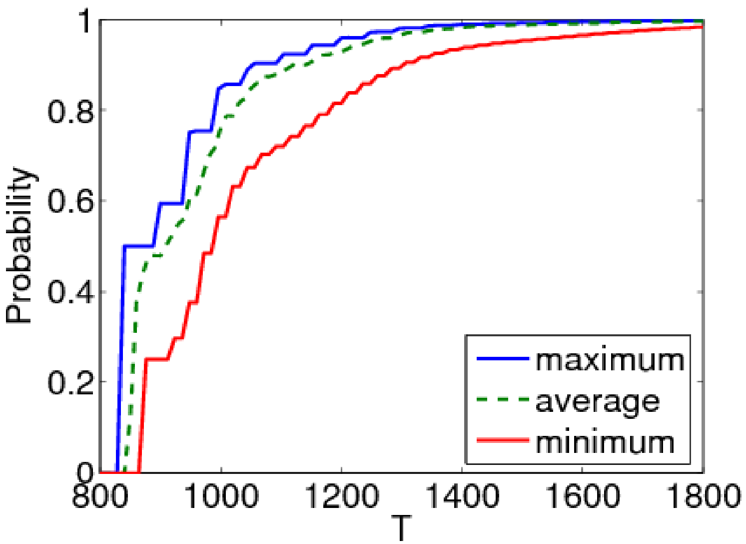
- Probabilistic reachability
  - fundamental concept in the quantitative verification of MDPs
  - $p_s^\sigma(F)$ = probability of reaching F starting from s under σ
  - consider the minimum/maximum values over all adversaries
  - $p_s^{min}(F) = \inf_\sigma p_s^\sigma(F)$ and $p_s^{max}(F) = \sup_\sigma p_s^\sigma(F)$

  - can be computed efficiently (and corresponding adversaries)

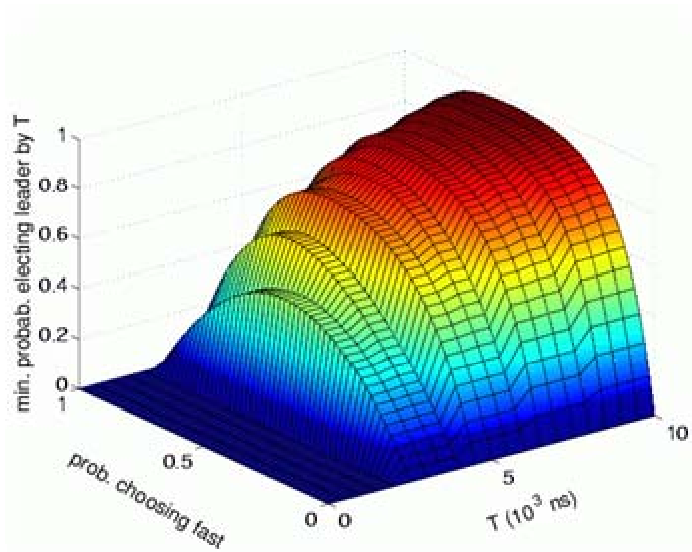
- Allows reasoning about best/worst-case behaviour
  - e.g. minimum probability of the protocol terminating correctly
  - e.g. maximum probability of a security breach

Probabilistic reachability for MDPs

- Often focus on quantitative properties:



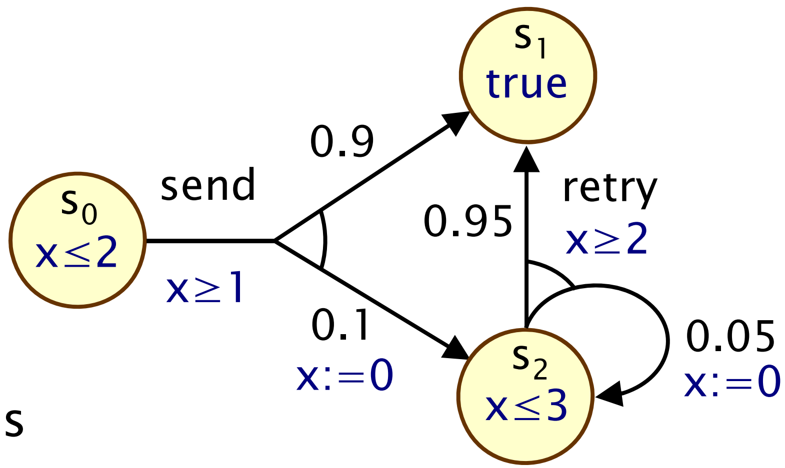
CSMA/CD network protocol:
Maximum, average and minimum probability that a message is sent successfully by time T



FireWire protocol:
Worst case (minimum) probability of electing a leader by time T for various coin biases

Probabilistic timed automata

- Probabilistic timed automata (PTAs)
  - Markov decision processes + real-valued clocks
  - or: timed automata + discrete probabilistic choice
  - models timed, probabilistic and nondeterministic behaviour
  - essential e.g. for communication protocols such as Zigbee, Bluetooth, which feature delays, randomisation, failures and concurrency



- PTA model checking
  - infinite-state MDP semantics
  - probabilistic (timed) reachability

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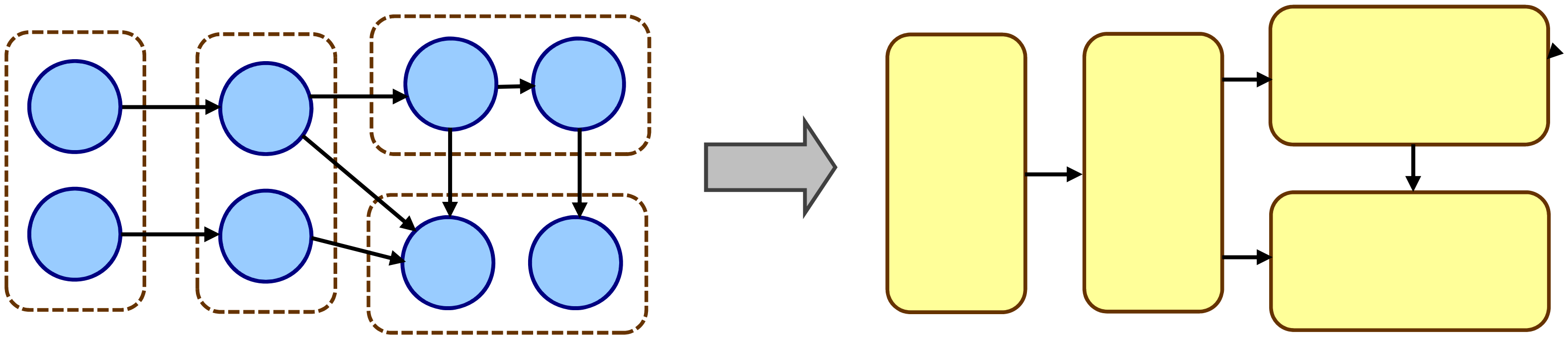
Abstraction

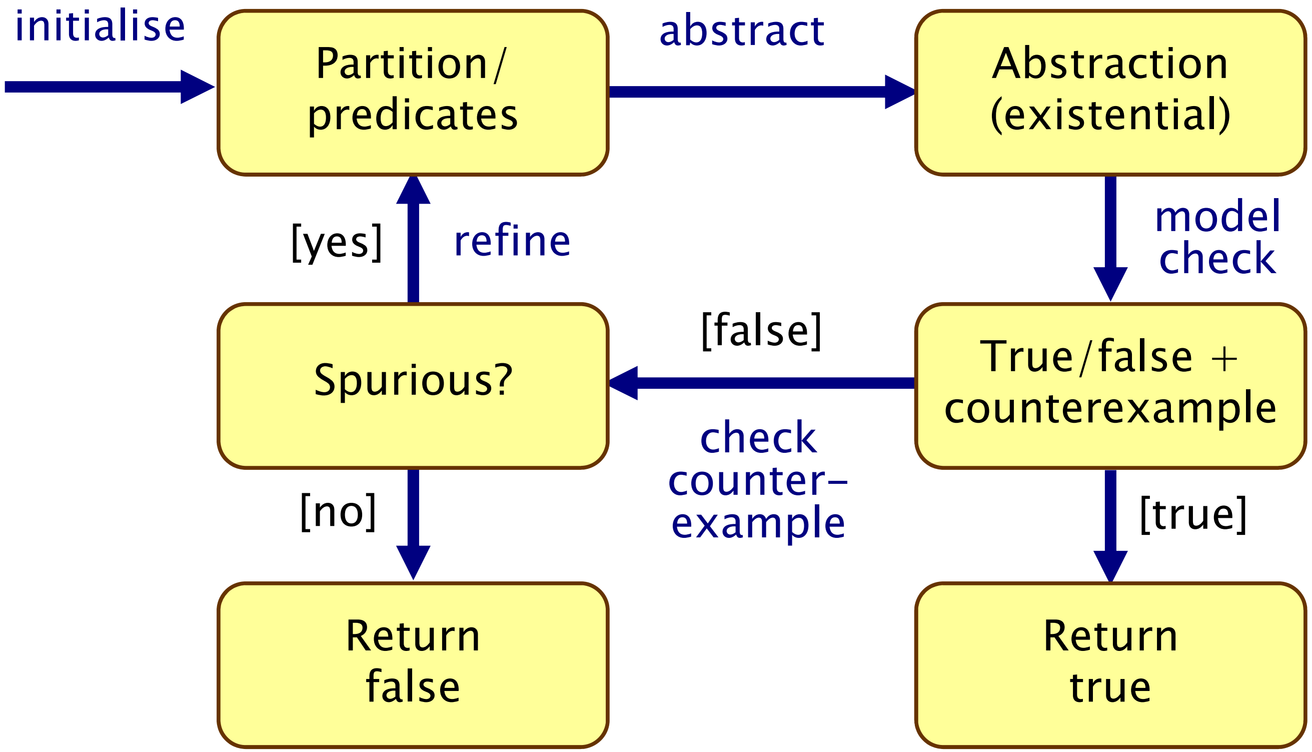
- Very successful in (non-probabilistic) formal methods
  - essential for verification of large/infinite-state systems
  - hide details irrelevant to the property of interest
  - yields smaller/finite model which is easier/feasible to verify
  - loss of precision: verification can return "don't know"

- Construct abstract model of a concrete system
  - e.g. based on a partition of the concrete state space
  - an abstract state represents a set of concrete states



Abstraction refinement (CEGAR)

- Counterexample-guided abstraction refinement
  - (non-probabilistic) model checking of reachability properties



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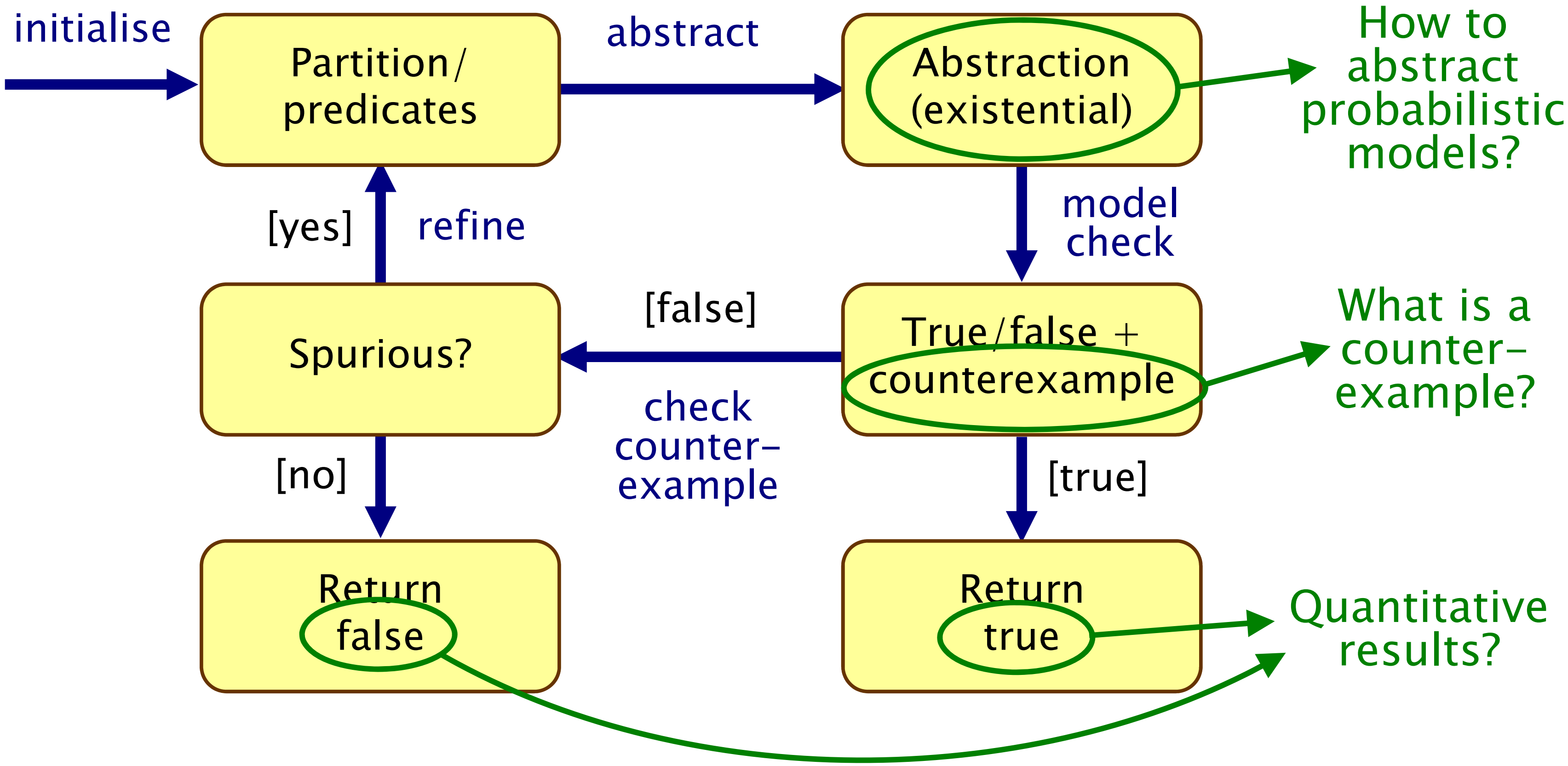


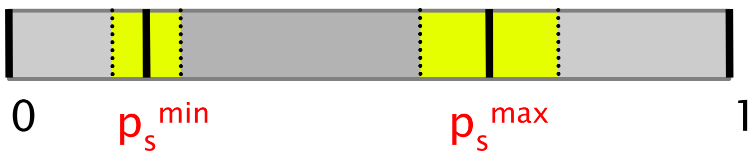
Abstraction of MDPs

- Abstraction increases degree of nondeterminism
  - i.e. minimum probabilities are lower and maximums higher

- But what form does the abstraction of an MDP take?

(i) an MDP [D'Argenio et al.'01]
  - probabilistic simulation relates concrete/abstract models

(ii) a stochastic two-player game [QEST'06]
  - separates nondeterminism from abstraction and from MDP
  - yields separate lower/upper bounds for min/max

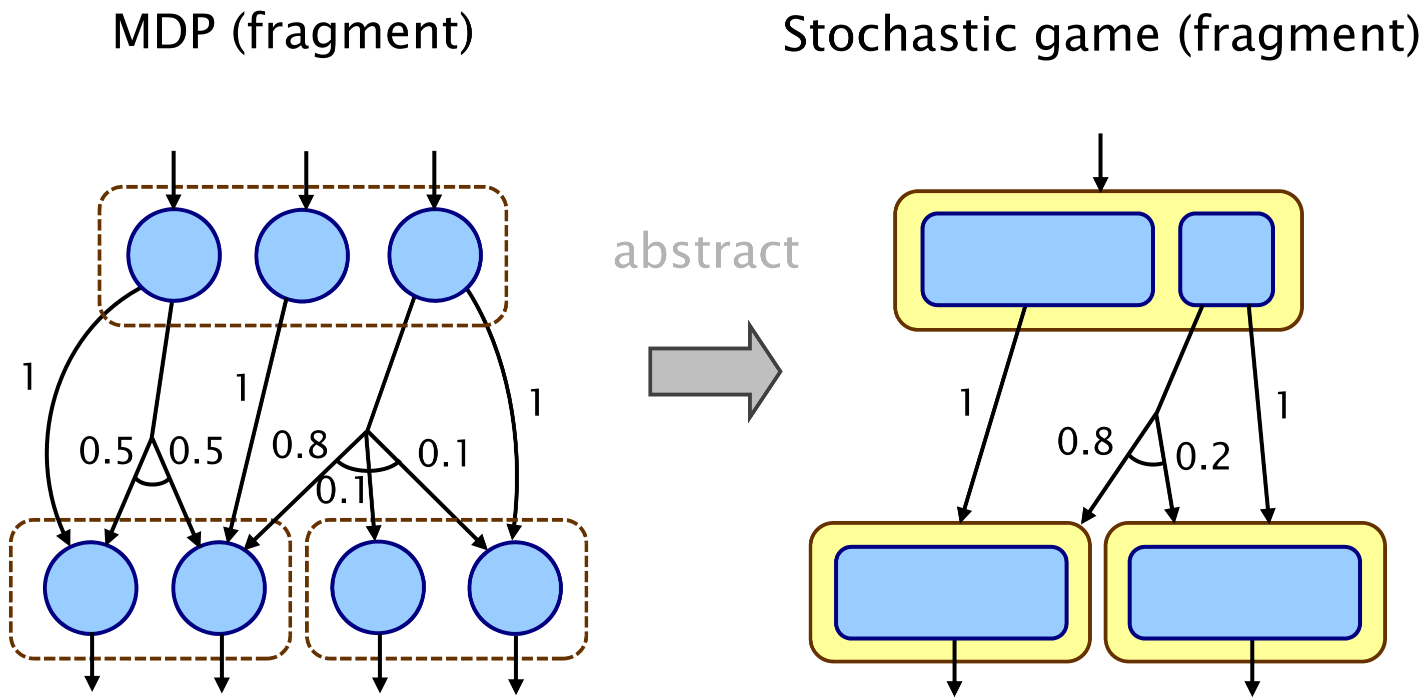


Stochastic two-player games

- Subclass of simple stochastic games [Shapley,Condon]
  - two nondeterministic players (1 and 2) and probabilistic choice
- Resolution of the nondeterminism in a game
  - corresponds to a pair of strategies for players 1 and 2: (σ_1,σ_2)
  - $p_a^{\sigma1,\sigma2}(F)$ probability of reaching F from a under (σ_1,σ_2)
  - can compute, e.g. : $\sup_{\sigma1} \inf_{\sigma2} p_a^{\sigma1,\sigma2}(F)$
  - informally: “the maximum probability of reaching F that player 1 can guarantee no matter what player 2 does”
- Abstraction of an MDP as a stochastic two-player game:
  - player 1 controls the nondeterminism of the abstraction
  - player 2 controls the nondeterminism of the MDP

Game abstraction (by example)

- Player $\boxed{1}$ vertices are partition elements (abstract states)
- (Sets of) distributions are lifted to the abstract state space
- States with same (sets of) choices form player $\boxed{2}$ vertices



Properties of the abstraction

- Analysis of game yields lower/upper bounds:
  - for target $F \in A$, $s \in S$ and $a \in A$ with $s \in a$

$$\inf_{\sigma 1,\sigma 2} p_a^{\sigma 1,\sigma 2}(F) \leq p_s^{\min}(F) \leq \sup_{\sigma 1} \inf_{\sigma 2} p_a^{\sigma 1,\sigma 2}(F)$$

$$\inf_{\sigma 1} \sup_{\sigma 2} p_a^{\sigma 1,\sigma 2}(F) \leq p_s^{\max}(F) \leq \sup_{\sigma 1,\sigma 2} p_a^{\sigma 1,\sigma 2}(F)$$

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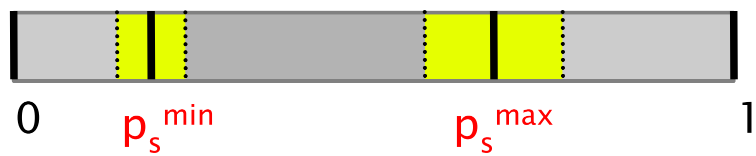
min/max reachability probabilities for original MDP



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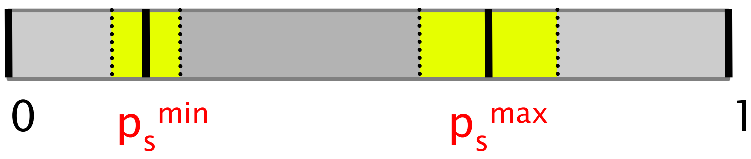
optimal probabilities for player 1, player 2 in game



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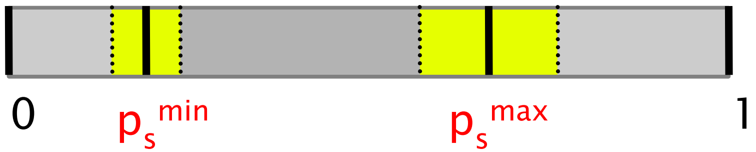
min/max reachability probabilities, treating game as MDP
(i.e. assuming that players 1 and 2 cooperate)

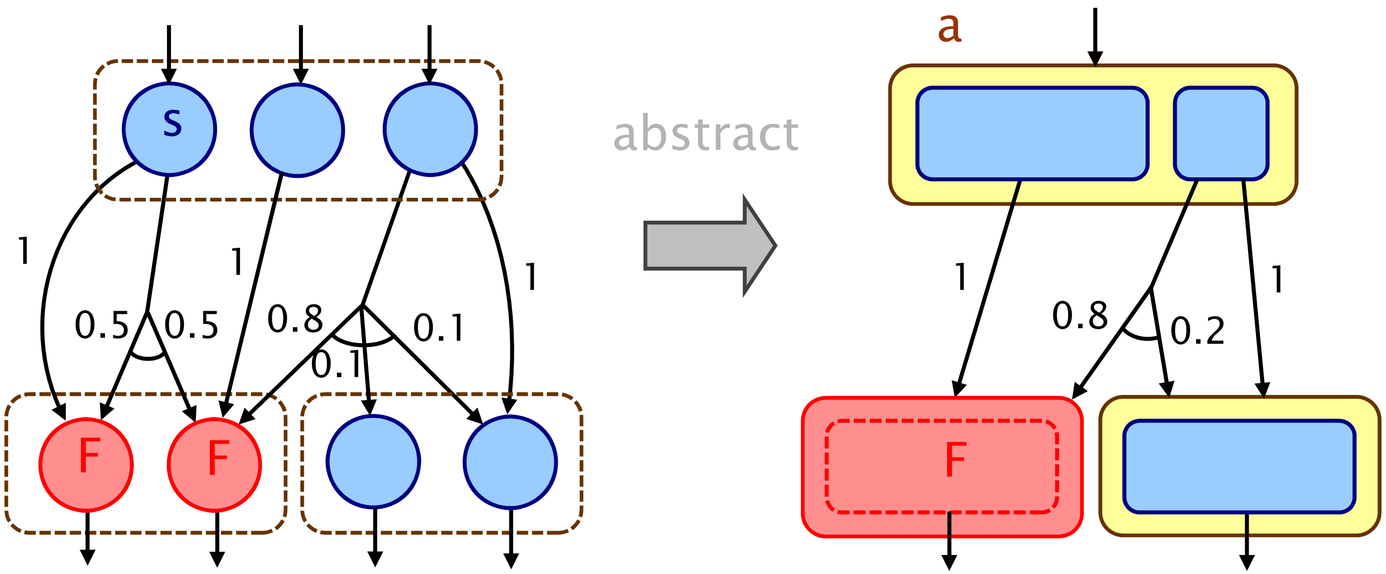


Example

$$\inf_{\sigma1} \sup_{\sigma2} p_a^{\sigma1,\sigma2}(F) = 0.8$$

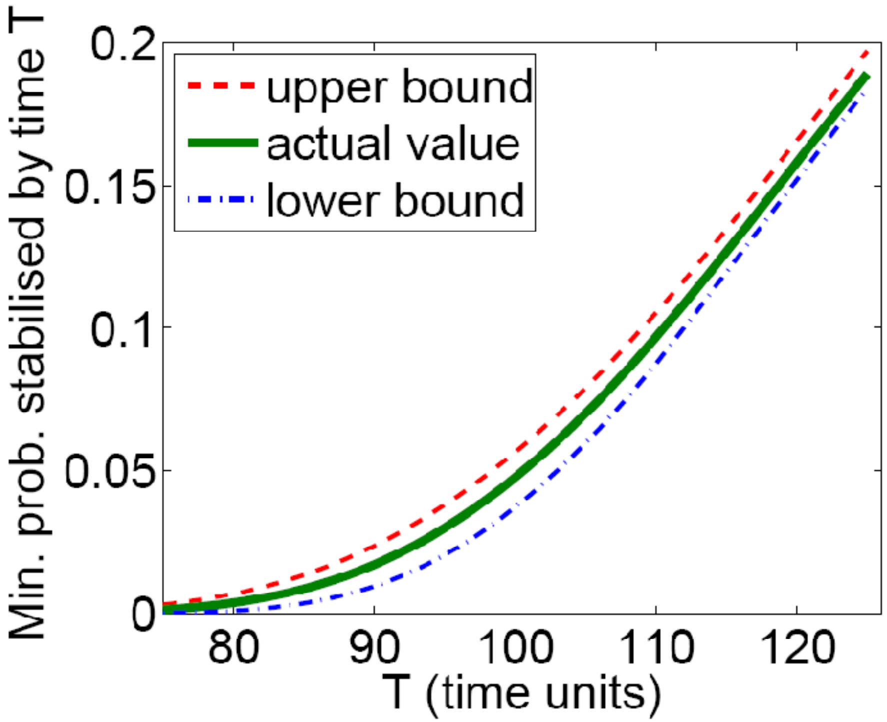
$$p_s^{max}(F) = 1 \in [0.8,1]$$

$$\sup_{\sigma1,\sigma2} p_a^{\sigma1,\sigma2}(F) = 1$$



Abstraction: Example results

- Israeli & Jalfon's Self Stabilisation [IJ90]
  - protocol for obtaining a stable state in a token ring
  - minimum probability of reaching a stable state by time T



concrete states: 1,048,575

abstract states: 627

Nondeterministic abstractions

- We can consider a general class of "nondeterministic" abstractions for probabilistic models

Concrete model:		Abstraction:
DTMC	→	MDP
MDP	→	STPG
CTMC	→	CTMDP
CTMDP	→	CTSTPG

- CTMDP = continuous-time Markov decision process
- CTSTPG = continuous-time stochastic two-player game

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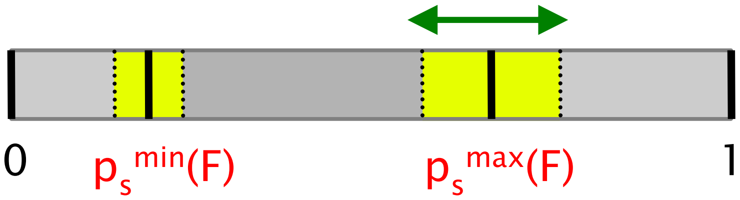
Abstraction refinement

- Consider (max) difference between lower/upper bounds
  - gives a quantitative measure of the abstraction's precision



- If the difference ("error") is too great, refine the abstraction
  - a finer partition yields a more precise abstraction
  - lower/upper bounds can tell us where to refine (which states)
  - (memoryless) strategies can tell us how to refine

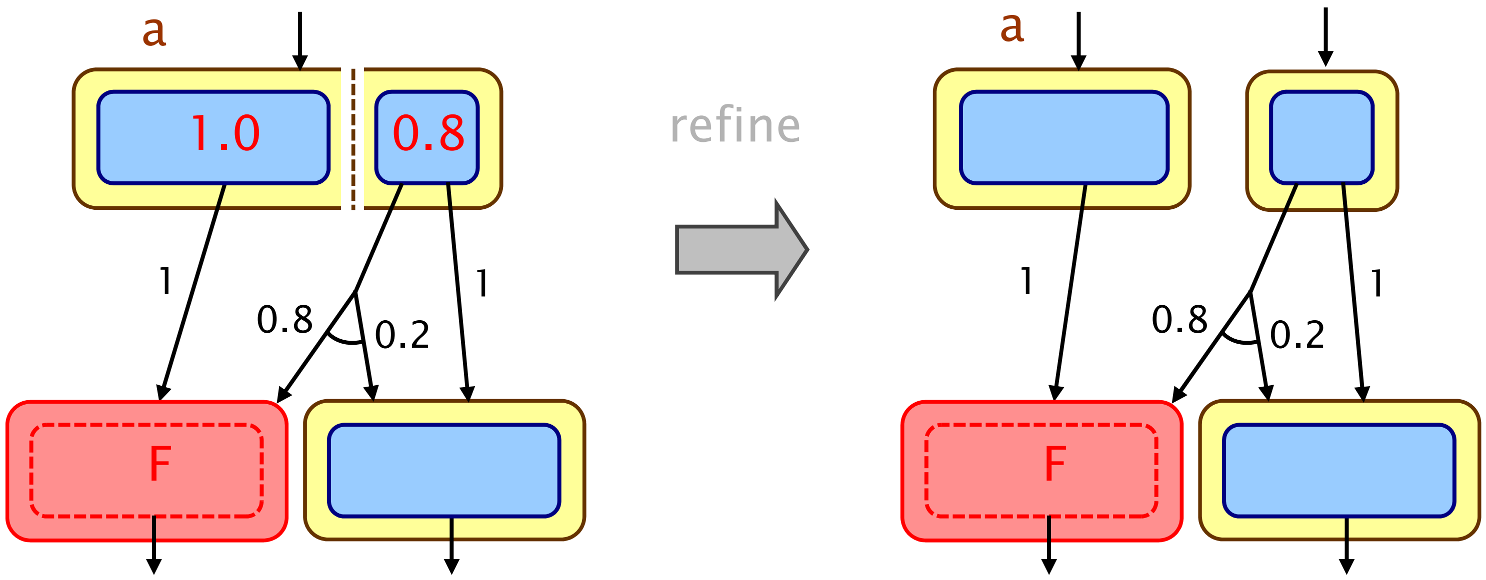
Example

$p_s^{max}(F) = 1 \in [0.8, 1]$

"error" $= 0.2$

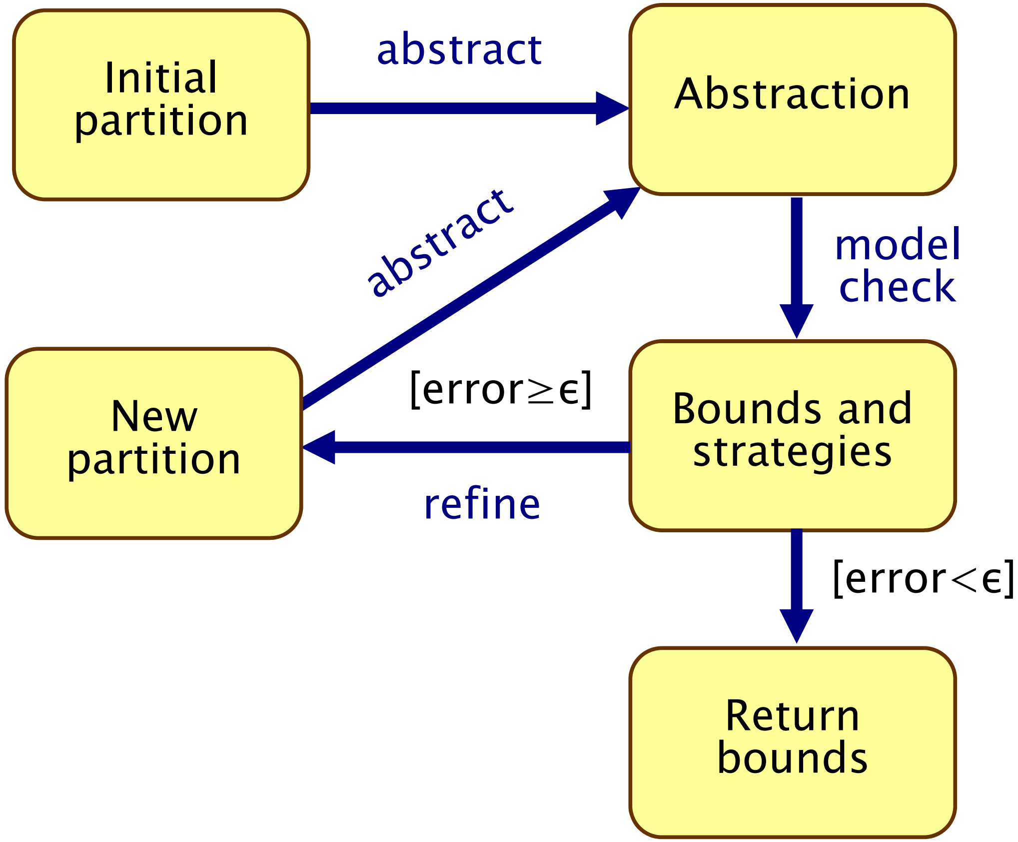
$p_s^{max}(F) = 1 \in [1, 1]$

"error" $= 0$



Abstraction-refinement loop

- Quantitative abstraction-refinement loop for MDPs



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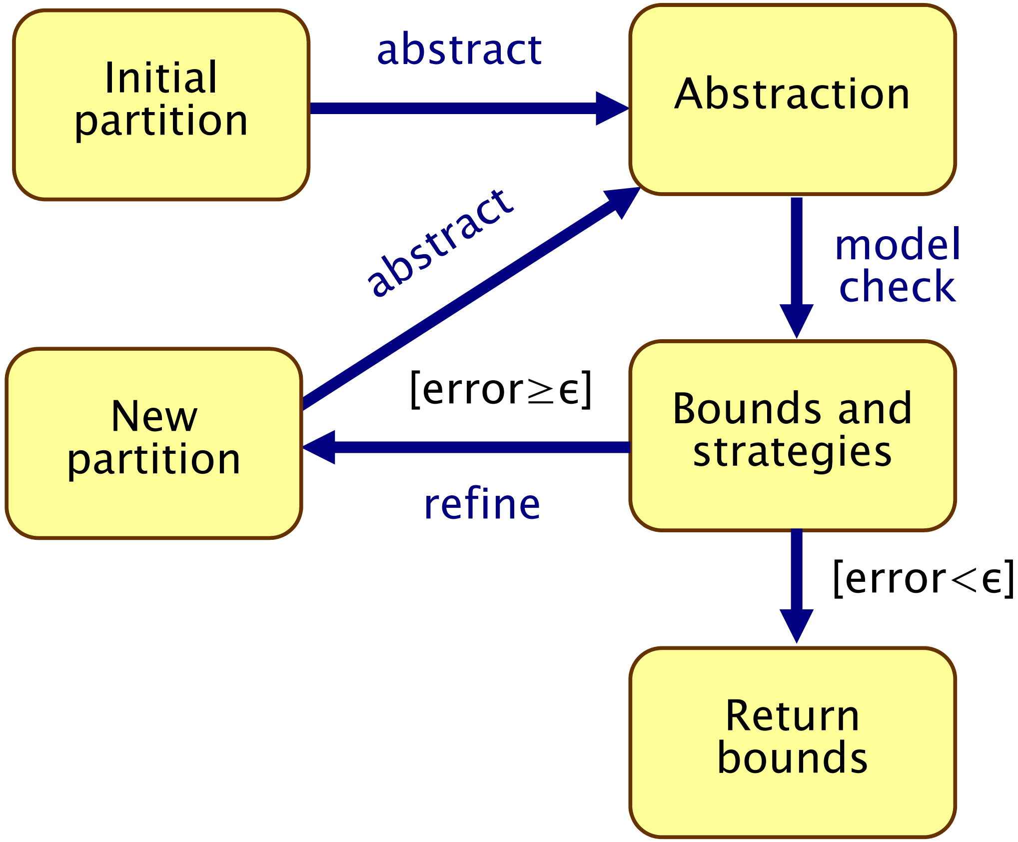


- Refinements yield strictly finer partition
- Guaranteed to converge for finite models
- Guaranteed to converge for infinite models with finite bisimulation

Abstraction-refinement loop

- Implementations of quantitative abstraction refinement…

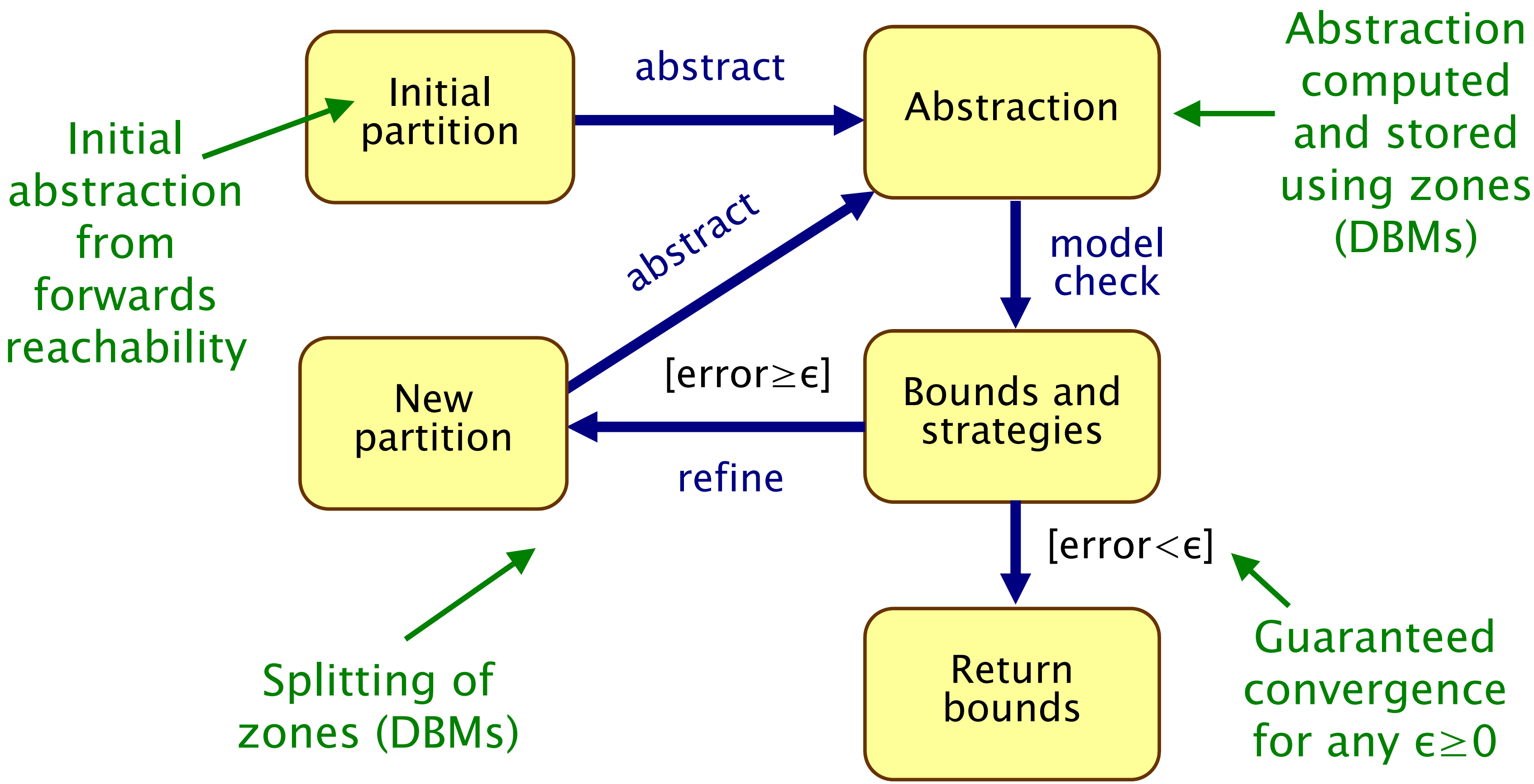
- Verification of probabilistic timed automata [FORMATS'09]
  - zone-based abstraction/refinement using DBMs
  - implemented in (next release of) PRISM
  - outperforms existing PTA verification techniques

- Verification of probabilistic software [VMCAI'09]
  - predicate abstraction/refinement using SAT solvers
  - implemented in tool qprover: components of PRISM, SATABS
  - analysed real network utilities (ping, tftp) - approx 1KLOC

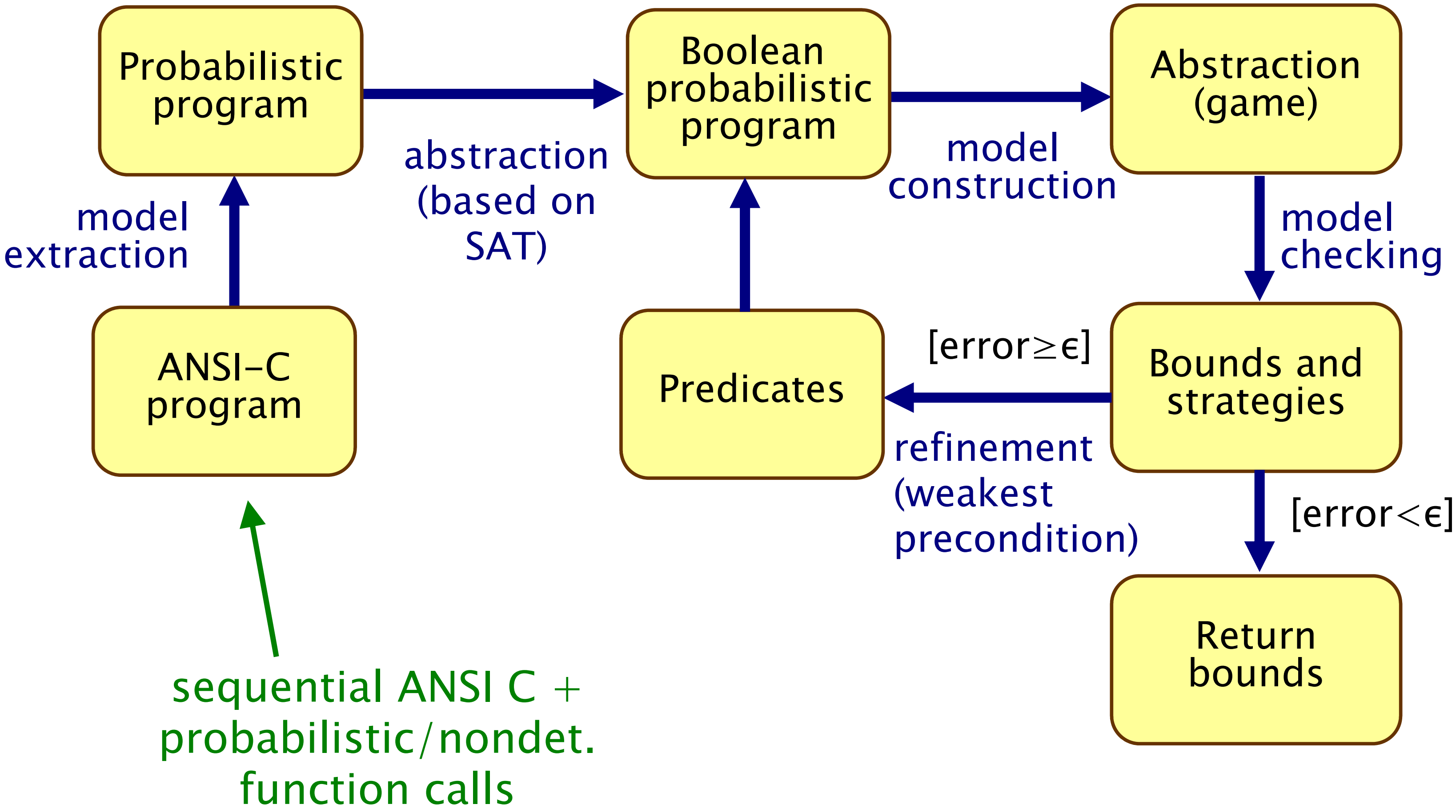
- Verification of concurrent PRISM models [Wachter/Zhang'10]
  - implemented in tool PASS; infinite-state PRISM models

Verification of PTAs

- Probabilistic model checking of PTAs



Verification of probabilistic software



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Related work

- Abstraction for Markov chains:
  - DTMCs: probability intervals (MDPs) [Fecher/Leucker/Wolf] [Huth]
  - CTMCs: using CTMDPs [Katoen/Klink/Leucker/Wolf]
  - CTMCs: sliding window abstraction [Henzinger/Mateescu/Wolf]
  - and more…
- Abstraction refinement for MDPs:
  - RAPTURE [D'Argenio/Jeannet/Jensen/Larsen]
  - probabilistic CEGAR [Hermanns/Wachter/Zhang]
  - magnifying lens abstraction [de Alfaro/Roy]
  - MDP-based abstractions [Chadha/Viswanathan]
  - and more…

Conclusions

- Abstraction for probabilistic models
  - MDPs (and PTAs) abstracted as stochastic two-player games
  - abstraction yields lower/upper bounds on probabilities
- Quantitative abstraction refinement
  - bounds give quantitative measure of utility of abstraction
  - bounds/strategies can be used to guide refinement
  - quantitative abstraction-refinement loop (for error $< \epsilon$)
  - fully automatic generation of abstraction
  - works in practice: probabilistic timed automata & software
- Current & future work
  - improved refinement heuristics, imprecise abstractions
  - software + time + probabilities
  - CTMCs, timed properties
  - probabilistic/stochastic hybrid systems